

# Can Two Non-congruent Shapes Have Equal Areas? Or, Can triangles look different but have equal areas?

In this *Geometer's Sketchpad* activity, you will sketch a triangle, measure its area, then change it so that its area equals the amount given to you. After that, you will copy it, and see if you can make a second non-congruent triangle (one with a totally different shape), but exactly the same area.

This activity assumes you already know how to use *Geometer's Sketchpad* to do the following:

- Sketch triangles using the “segment” tool,
- Measure lengths of segments,
- Label (and re-name) points,
- Construct a triangle interior, and
- Marquee select a group of objects.



- Sketch a triangle that if cut from paper would be no larger than the palm of your hand.
- Label the three vertices as A, B, and C (re-name if needed).
- Measure the lengths of all three sides of triangle ABC (in symbols:  $\Delta ABC$ ); slide them beside the sides they measure.
- If you don't already have a scalene  $\Delta$ , move a vertex or two until you have a scalene  $\Delta$ .
- Construct an interior for  $\Delta ABC$ ; use whatever color you want.
- De-select everything before going to step 2.



- Select the interior of your triangle (when you click on it, it will show crossed-hatched lines)
- In the menu bar, click Measure > Area
- Move the measurement of the area to be above the triangle: remember to de-select it.
- Click on any vertex of  $\Delta ABC$ : move it around until  $\Delta ABC$  has an area of exactly 10.00  $\text{cm}^2$ .



- Marquee select  $\Delta ABC$  and all of its measurements, including the area.
- Copy all of it (Edit > Copy or CTRL+C)
- Paste (Edit > Paste or CTRL+V) - a duplicate will appear, highlighted, overlapping
- Carefully click on the pasted copy and drag it to the right of  $\Delta ABC$ .  
**Hint:** click and drag using a highlighted measurement - this is a large, easy-to-grab object. If you miss and lose the highlighting, undo your work (CTRL+Z), paste again.
- De-select everything.



- Re-name the vertices of your second triangle so it becomes  $\Delta DEF$ .
- Move any of the vertices of  $\Delta DEF$  so that  $\Delta DEF$  has a totally new shape, different side measurements, BUT still has an area of exactly 10.00  $\text{cm}^2$ .
- Repeat step 3 to produce a third triangle,  $\Delta GHJ$ , that is totally different in shape and lengths, but which also has an area of exactly 10.00  $\text{cm}^2$ .



- Change  $\Delta DEF$  to become an isosceles triangle, with an area of exactly 10.00  $\text{cm}^2$ .
- Change  $\Delta GHJ$  to become an equilateral triangle, with an area of exactly 10.00  $\text{cm}^2$ .
- Repeat steps above, making  $\Delta KLM$ , a right triangle with an area of exactly 10.00  $\text{cm}^2$ , and  $\Delta NPQ$ , an obtuse triangle with an area of exactly 10.00  $\text{cm}^2$ .